

Pulsating magneto-dipole radiation of a quaking neutron star powered by energy of Alfvén seismic vibrations

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Abstract We compute the characteristic parameters of magnetic dipole radiation of a neutron star undergoing torsional seismic vibrations under the action of Lorentz restoring force about axis of a dipolar magnetic field experiencing decay. After brief outline of general theoretical background of the model of vibration powered neutron star, we present numerical estimates of basic vibration and radiation characteristics, such as the oscillation frequency, lifetime, luminosity of radiation, and investigate their time dependence upon magnetic field decay. The presented analysis suggests that gradual decrease in frequencies of pulsating high-energy emission detected from a handful of currently monitored AXP/SGR-like X-ray sources can be explained as being produced by vibration powered magneto-dipole radiation of quaking magnetars.

Key words: neutron stars, torsion Alfvén vibrations, vibration powered radiation, magnetic field decay, magnetars

1 INTRODUCTION

The last two decades have seen an increasing interest in the Soft Gamma Repeaters (SGRs) and Anomalous X-ray Pulsars – commonly referred to as magnetars (Duncan & Thompson 1992), the seismic and radiative activity of which is fairly different from that of rotation powered radio pulsars (e.g. Kouveliotou 1999, Harding 1999, Woods & Thompson 2006, Mereghetti 2008, Qiao, Xu & Du 2010). The most popular idea is that energy supply of long-periodic pulsating radiation of these fairly young neutron stars comes from process involving decay of ultra strong magnetic field. One of such processes could be magneto-mechanical vibrations driven by forces of magnetic-field-dependent stresses (Bastrukov et al. 2002). In the development of this line of argument, particular attention has been given to torsion vibrations of perfectly conducting stellar matter about magnetic axis of the star under the action of magnetic Lorentz force with focus on discrete frequency spectra of toroidal Alfvén mode (a -mode). In the past, the standing-wave regime of such vibrations has been subject of several investigations (Ledoux & Walraven 1958). In works (Bastrukov et al. 2009a, 2009b, 2010) focus was made on the non-investigated regime of node-free vibrations in the static (time-independent) field. The prime purpose of these latter works was to get some insight into difference between spectra of discrete frequencies of toroidal a -modes in neutron star models having one and the same mass M and radius R , but different shapes of constant-in-time poloidal magnetic fields. By use of the Rayleigh energy method, it was found that each specific form of spatial configuration of static magnetic field about axis of which the neutron star matter undergoes node-free differentially rotational oscillations is uniquely reflected in the discrete

frequency spectra by form of dependence of frequency upon overtone ℓ of vibrations. The subject of present our study is radiative activity of quaking neutron star powered by energy of Alfvén vibrations in its own magnetic field experiencing decay. Part of this project has been reported in recent workshops and conferences and in short paper (Bastrukov et al. 2011). Taking this into account, in this article only a brief overview of theoretical background of the model is given. The focus is placed on numerical computation of basic characteristics of vibration and radiation which are of interest in observational search for such objects.

2 GENERAL BACKGROUND OF THE VIBRATION POWERED NEUTRON STAR MODEL

Only a brief outline of this model is given here and more details can be found elsewhere (Bastrukov et al. 2011). The Lorentz-force-driven differentially rotational vibrations of perfectly conducting matter of neutron star about axis of poloidal internal and dipolar external magnetic field evolving in time are properly described in terms of material displacements \mathbf{u} obeying equation of magneto-solid-mechanics

$$\rho(r) \ddot{\mathbf{u}}(\mathbf{r}, t) = \frac{1}{4\pi} [\nabla \times [\nabla \times [\mathbf{u}(\mathbf{r}, t) \times \mathbf{B}(\mathbf{r}, t)]] \times \mathbf{B}(\mathbf{r}, t) \quad (1)$$

$$\dot{\mathbf{u}}(\mathbf{r}, t) = [\boldsymbol{\omega}(\mathbf{r}, t) \times \mathbf{r}], \quad \boldsymbol{\omega}(\mathbf{r}, t) = A_t [\nabla \chi(r)] \dot{\alpha}(t). \quad (2)$$

The field $\dot{\mathbf{u}}(\mathbf{r}, t)$ is identical to that for torsion node-free vibrations restored by Hooke's force of elastic stresses (Bastrukov et al. 2007, 2010) with $\chi(\mathbf{r}) = A_\ell f_\ell(r) P_\ell(\cos \theta)$ where $f_\ell(r)$ is the nodeless function of distance from the star center and $P_\ell(\cos \theta)$ is Legendre polynomial of degree ℓ specifying the overtone of toroidal mode. In (2), the amplitude $\alpha(t)$ is the basic dynamical variable describing time evolution of vibrations which is different for each specific overtone. The bulk density can be represented in the form $\rho(r) = \rho \phi(r)$, where ρ is the density at the star center and $\phi(r)$ describes the radial profile of density which can be taken from computations of neutron star structure relying on realistic equations of state accounting for non-uniform mass distribution in the star interior (e.g., Weber 1999). The central to the subject of our study is the following representation of the time-evolving internal magnetic field $\mathbf{B}(\mathbf{r}, t) = B(t) \mathbf{b}(\mathbf{r})$, where $B(t)$ is the time-dependent intensity and $\mathbf{b}(\mathbf{r})$ is dimensionless vector-function of the field distribution over the star volume. The gist of the energy variational method of computing frequency of nod-free Alfvén vibrations consists in the following separable representation of material displacements $\mathbf{u}(\mathbf{r}, t) = \mathbf{a}(\mathbf{r}) \alpha(t)$. Scalar product of (??) with this form of \mathbf{u} and integration over the star volume leads to equation for amplitude $\alpha(t)$ having the form of equation of oscillator with time-depended spring constant

$$\mathcal{M} \ddot{\alpha}(t) + \mathcal{K}(t) \alpha(t) = 0, \quad (3)$$

$$\mathcal{M} = \rho m, \quad m = \int \phi(r) \mathbf{a}(\mathbf{r}) \cdot \mathbf{a}(\mathbf{r}) d\mathcal{V}, \quad \mathbf{a} = A_t \nabla \times [\mathbf{r} f_\ell(r) P_\ell(\cos \theta)]$$

$$\mathcal{K} = \frac{B^2(t)}{4\pi} k, \quad k = \int \mathbf{a}(\mathbf{r}) \cdot [\mathbf{b}(\mathbf{r}) \times [\nabla \times [\nabla \times [\mathbf{a}(\mathbf{r}) \times \mathbf{b}(\mathbf{r})]]]] d\mathcal{V}$$

As for the general asteroseismology of compact objects is concerned, the above equations seems to be appropriate not only for neutron stars but also white dwarfs (Molodtsova et al 2010, Bastrukov et al 2010a) and quark stars. The superdense material of strange quark stars is too expected to be in solid state (Xu 2003, 2009). In Fig.1 we plot the fundamental frequency $\nu_A = \omega_A/2\pi$ (where $\omega_A = v_A/R$) and the period $P_A = \nu_A^{-1}$ of global Alfvén oscillations

$$\nu_A = \frac{B}{2\pi} \sqrt{\frac{R}{3M}}, \quad P_A = \frac{2\pi}{B} \sqrt{\frac{3M}{R}} \quad (4)$$

as functions of intensity of constant in time magnetic field B for solid star models with masses and radii of typical white dwarfs, neutron stars and quark stars.

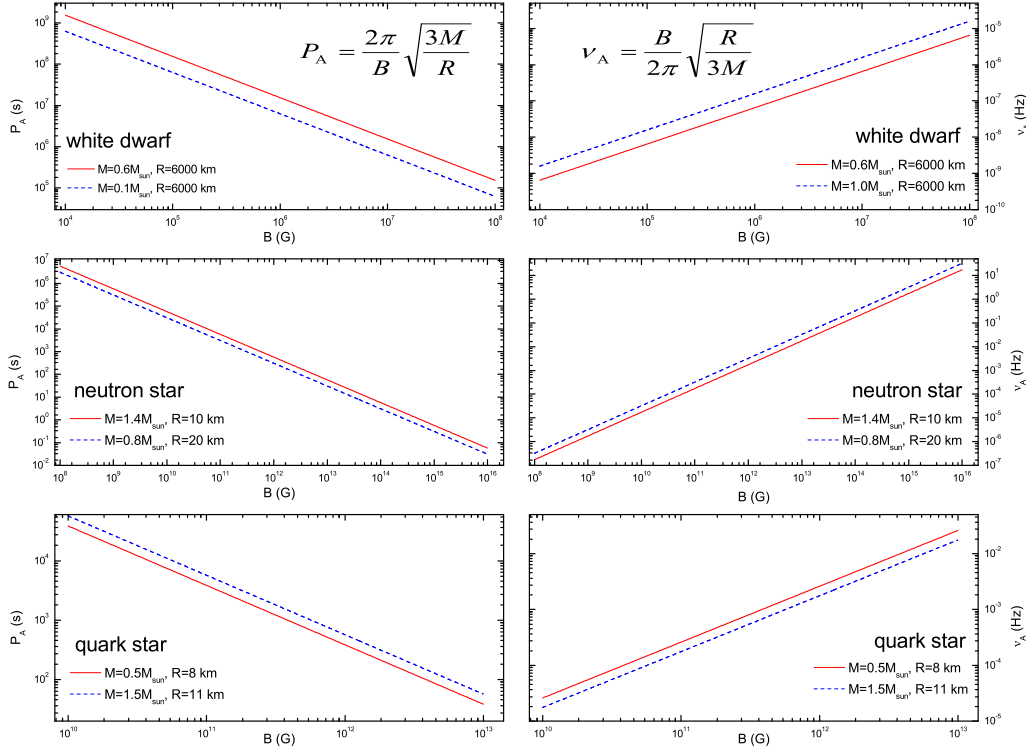


Fig. 1 The basic frequencies and periods of global Alfvén oscillations as functions of magnetic field for the typical mass and radius of white dwarfs, neutron stars and quark stars. The difference in mass and radius can be regarded as reflecting difference in underlying equations of state of matter in these compact objects.

The total vibration energy and frequency are given by

$$E_A(t) = \frac{\mathcal{M}\dot{\alpha}^2(t)}{2} + \frac{\mathcal{K}(B(t))\alpha^2(t)}{2}, \quad \omega(t) = \sqrt{\frac{\mathcal{K}(t)}{\mathcal{M}}} = B(t)\kappa, \quad \kappa = \sqrt{\frac{R}{3M}} s. \quad (5)$$

Here M and R are the neutron star mass and radius and s is the parameter depending on overtone of Alfvén toroidal mode and the depth of seismogenic layer. The energy conversion of above magneto-mechanical vibrations into magneto-dipole radiation which is governed by equation

$$\frac{dE_A(t)}{dt} = -\mathcal{P}(t), \quad \mathcal{P}(t) = \frac{2}{3c^3} \delta\ddot{\mu}^2(t). \quad (6)$$

The axisymmetric torsional oscillations of matter around magnetic axis of the star are accompanied by fluctuations of total magnetic moment preserving its initial (in seismically quiescent state) direction: $\boldsymbol{\mu} = \mu \mathbf{n} = \text{constant}$. The frequency $\omega(t)$ of such oscillations must be the same for both fluctuations of magnetic momenta $\delta\boldsymbol{\mu}(t)$ and above magneto-mechanical oscillations which are described in terms of $\alpha(t)$, namely

$$\delta\ddot{\mu}(t) + \omega^2(t)\delta\mu(t) = 0, \quad \ddot{\alpha}(t) + \omega^2(t)\alpha(t) = 0, \quad \omega^2(t) = B^2(t)\kappa^2. \quad (7)$$

This suggests $\delta\boldsymbol{\mu}(t) = \boldsymbol{\mu}\alpha(t)$. On account of this the equation of energy conversion is reduced to the following law of magnetic field decay

$$\frac{dB(t)}{dt} = -\gamma B^3(t), \quad \gamma = \frac{2\mu^2\kappa^2}{3\mathcal{M}c^3} = \text{constant}, \quad (8)$$

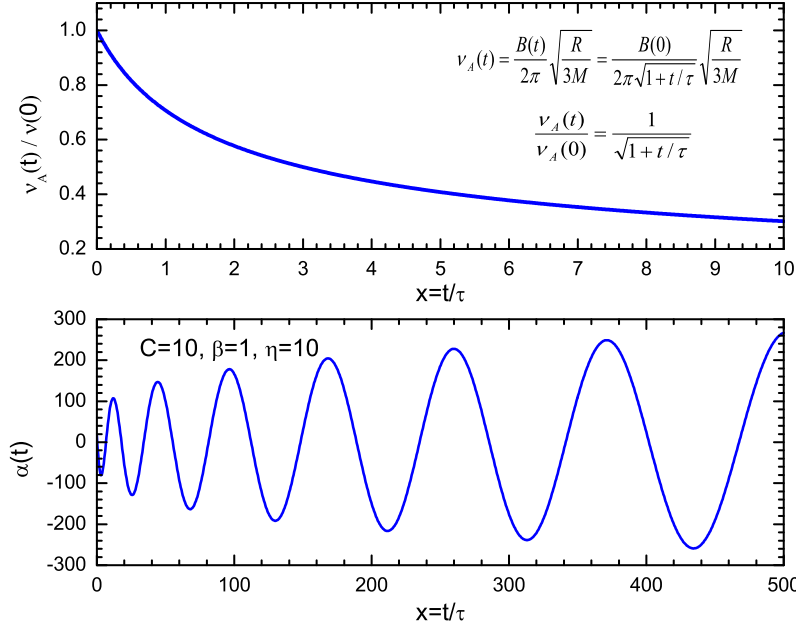


Fig. 2 The figure illustrates the effect of magnetic field decay on the vibration frequency and amplitude of quadrupole toroidal a -mode presented as functions of $x = t/\tau$.

$$B(t) = \frac{B(0)}{\sqrt{1+t/\tau}}, \quad \tau^{-1} = 2\gamma B^2(0). \quad (9)$$

Thus, the magnetic field decay resulting in the loss of total energy of Alfvén vibrations of the star causes its vibration period $P \sim B^{-1}$ to lengthen at a rate proportional to the rate of magnetic field decay. The time evolution of vibration amplitude (the solution of equation for $\alpha(t)$, obeying two conditions $\alpha(t=0) = \alpha_0$ and $\alpha(t=\tau) = 0$) reads¹(e.g., Polyanin & Zaitzev 2004)

$$\alpha(t) = C s^{1/2} \{J_1(z(t)) - \eta Y_1(z(t))\}, \quad z(t) = 2\beta s^{1/2}(t), \quad s = 1 + t/\tau \quad (10)$$

where $J_1(z)$ and $Y_1(z)$ are Bessel functions (e.g., Abramowitz & Stegun 1972) and

$$\alpha_0^2 = \frac{2\bar{E}_A(0)}{M\omega^2(0)} = \frac{2\bar{E}_A(0)}{K(0)}, \quad \omega^2(0) = \frac{K(0)}{M}. \quad (11)$$

Here by $\bar{E}_A(0)$ is understood the average energy stored in torsional Alfvén vibrations of magnetar. If all the detected energy E_{burst} of X-ray outburst goes in the quake-induced vibrations, $E_{\text{burst}} = E_A$, then the initial amplitude α_0 is determined unambiguously. Thus, the magnetic field decay is crucial to the energy conversion from Alfvén vibrations to electromagnetic radiation whose most striking feature is the gradual decrease of frequency of pulses (period lengthening). The magnetic-field-decay induced lengthening of period of pulsating radiation (equal to period of vibrations) is described by

$$P(t) = P(0) [1 + (t/\tau)]^{1/2}, \quad \dot{P}(t) = \frac{1}{2\tau} \frac{P(0)}{[1 + (t/\tau)]^{1/2}},$$

$$\tau = \frac{P^2(0)}{2P(t)\dot{P}(t)}, \quad P(0) = \frac{2\pi}{\kappa B(0)} \quad (12)$$

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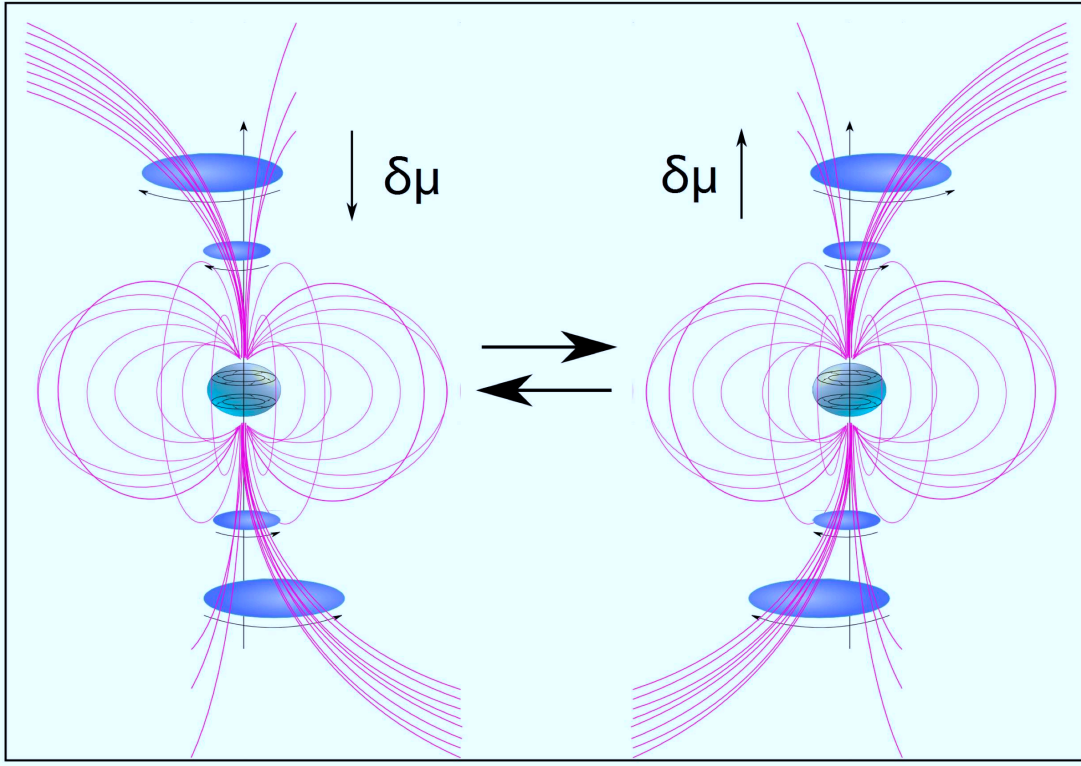


Fig. 3 Artist view of quadrupole overtone of torsional Alfvén vibrations of magnetar about magnetic axis producing oscillations of lines of dipolar magnetic field defining the beam direction of outburst X-ray emission.

On comparing τ given by equations (9) and (12), one finds that interrelation between equilibrium equilibrium value of the total magnetic moment μ of a neutron star of mass $M = 1.2 M_{\odot}$ and radius $R = 15$ km vibrating in quadrupole overtone of toroidal a -mode, pictured in Fig.3, is given by

$$\mu = 5.5 \times 10^{37} \sqrt{P(t) \dot{P}(t)}, \text{ G cm}^3. \quad (13)$$

This illustrative picture shows that pulsating radiation in question belongs to the periodic changes of polarization of magneto-dipole radiation powered by Alfvén seismic vibrations of neutron star. In the reminder we present numerical estimates of characteristic parameters of of pulsating magneto-dipole radiation produced by quaking neutron star at the expense of energy of Lorentz-force driven differentially rotational vibrations about axis of dipole magnetic moment.

3 NUMERICAL ANALYSIS

To get an idea of the magnitude of characteristic parameter of vibrations providing energy supply of magneto-dipole radiation of a neutron star, in Table 1 we present results of numerical computations of the fundamental frequency of neutron star oscillations in quadrupole toroidal a -mode and time of decay of magnetic field τ as functions of increasing magnetic field.

As was emphasized, the most striking feature of considered model of vibration powered radiation is the lengthening of periods of pulsating emission caused by decay of internal magnetic field. This

Table 1 The Alfvén frequency of Lorentz-force-driven torsion vibrations, ν_A , and their life-time equal to decay time of magnetic field, τ , in neutron stars with magnetic fields typical for pulsars and magnetars.

	$M(M_\odot)$	$R(\text{km})$	$B(\text{G})$	$\nu_A(\text{Hz})$	$\tau(\text{yr})$
Pulsars	0.8	20	10^{12}	3.25×10^{-3}	4.53×10^{10}
	1.0	15	10^{13}	2.52×10^{-2}	2.98×10^7
Magnetars	1.1	13	10^{14}	0.22	7.4×10^3
	1.2	12	10^{15}	2.06	1.31
	1.3	11	10^{15}	1.89	2.38
	1.4	10	10^{16}	17.4	4.44×10^{-4}

suggests that this model is relevant to electromagnetic activity of magnetars - neutron stars endowed with magnetic field of extremely high intensity the radiative activity of which is ultimately related to the magnetic field decay. Such a view is substantiated by estimates of Alfvén frequency presented in the table. For magnetic fields of typical rotation powered radio pulsars, $B \sim 10^{12}$ G, the computed frequency ν_A is much smaller than the detected frequency of pulses whose origin is attributed to lighthouse effect. In the meantime, for neutron stars with magnetic fields $B \sim 10^{14}$ G the estimates of ν_A are in the realm of observed frequencies of high-energy pulsating emission of soft gamma repeaters (SGRs), anomalous X-ray pulsars (AXPs) and sources exhibiting similar features. According to common belief, these are magnetars - highly magnetized neutron stars whose radiative activity is related with magnetic field decay (e.g., Woods & Thompson 2006). The amplitude of vibrations is estimated as

$$\alpha_0 = \left[\frac{2\bar{E}_A(0)}{\mathcal{M}\omega^2(0)} \right]^{1/2} = 3.423 \times 10^{-3} \bar{E}_{A,40}^{1/2} B_{14}^{-1} R_6^{-3/2}. \quad (14)$$

where $\bar{E}_{A,40} = \bar{E}_A/(10^{40} \text{ erg})$ is the energy stored in the vibrations. $R_6 = R/(10^6 \text{ cm})$ and $B_{14} = B/(10^{14} \text{ G})$. The presented computations show that the decay time of magnetic field (equal to duration time of vibration powered radiation in question) strongly depends on the intensity of initial magnetic field of the star: the larger magnetic field B the shorter time of radiation τ at the expense of energy of vibration in decay during this time magnetic field. The effect of equation of state of neutron star matter (which is most strongly manifested in different values mass and radius of the star) on frequency ν_A is demonstrated by numerical vales of this quantity for magnetars with one and the same value of magnetic field $B = 10^{15}$ G but different values of mass and radius.

According to above presented analytic results, the neutron star luminosity powered by neutron star vibrations in quadrupole toroidal a - is given by

$$\mathcal{P} = \frac{\mu^2}{c^3} \kappa^4 B^4(t) \alpha^2(t), \quad \mu = (1/2)B(0)R^3, \quad B(t) = B(0)[1 + t/\tau]^{-1/2}, \quad (15)$$

$$\alpha(t) = C s^{1/2} \{J_1(z(t)) - \eta Y_1(z(t))\}, \quad z = 2\omega(0)\tau(1 + t/\tau) \quad (16)$$

The presented in Fig.4 computations of power of magneto-dipole radiation of a neutron stars (of one and the same mass and radius but different values of magnetic fields) exhibit oscillating character of luminosity. The frequency of these oscillations equal to that of torsional Alfvén seismic vibrations of neutron star. The practical usefulness of presented computations is that they can be used as a guide in observational quest of vibration powered neutron stars among the currently monitoring AXP/SGR-like sources.

4 SUMMARY

It is generally realized today that the standard model of inclined rotator, lying at the base of our understanding of radio pulsars, faces serious difficulties in explaining the long-periodic ($2 < P < 12$ s)

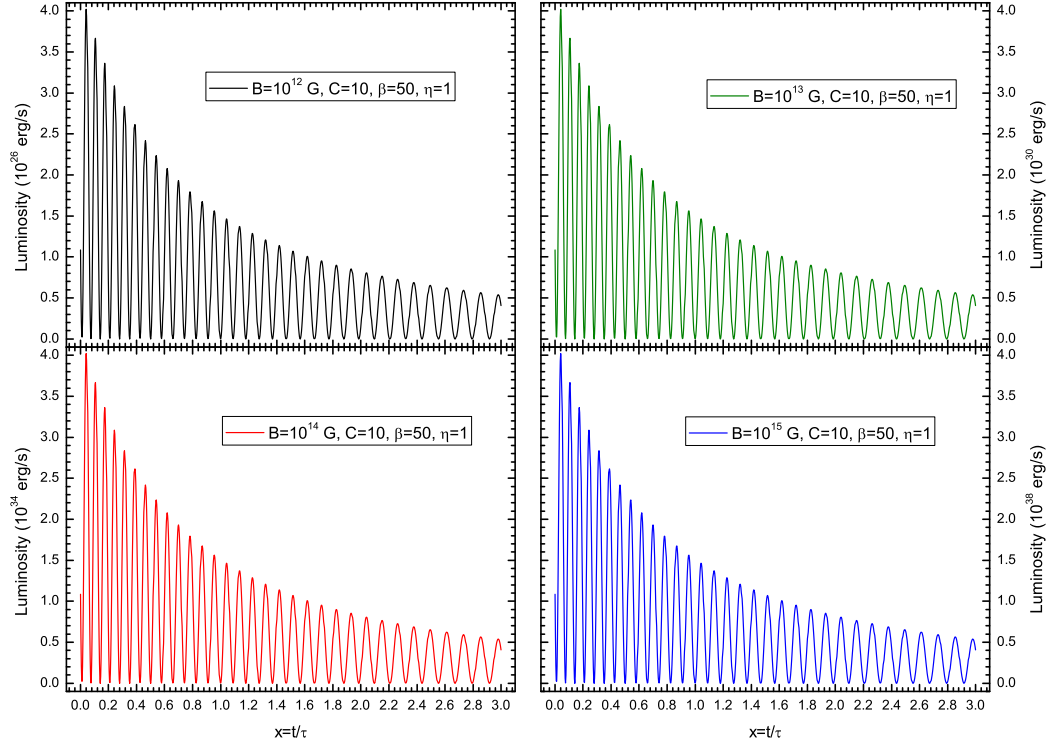


Fig. 4 Time evolution of luminosity of magneto-dipole radiation powered by energy of torsional Alfvén seismic vibrations of neutron star with mass $M = 1.2M_{\odot}$ and radius $R = 15$ km with intensities of magnetic field.

pulsed radiation of soft gamma repeaters (SGRs) and anomalous X -ray pulsars (AXPs). Observations show that persistent X -ray luminosity of these sources ($10^{34} < L_X < 10^{36} \text{ erg s}^{-1}$) is appreciably (10-100 times) larger than expected from neutron star deriving radiation power from energy of rotation with frequency of detected pulses. It is believed that this discrepancy can be resolved assuming that AXP/SGR-like sources are magnetars – young, isolated and seismically active neutron stars whose energy supply of pulsating high-energy radiation comes not from rotation (as is the case of radio pulsars) but from different process involving decay of ultra strong magnetic field, $10^{14} < B < 10^{16} \text{ G}$. Adhering to this attitude we have investigated the model of neutron star deriving radiation power from the energy of torsional Lorentz-force-driven oscillations in its own magnetic field experiencing decay. It worth noting that such an idea is not new and first has been discussed by Hoyle et al. (1964), still before the discovery of pulsars (e.g., Pacini 2008). What is newly disclosed here is that such radiation is possible when magnetic field is decayed. Since the magnetic field decay is one of the most conspicuous features distinguishing magnetars from rotation powered pulsars, it seems meaningful to expect that at least some of AXP/SGR - like sources can be vibration powered magnetars. Working from this, we have presented extended numerical analysis of the model of vibration powered neutron star whose results can be efficiently, as is hoped, utilized as a guide for discrimination of vibration powered from rotation powered neutron stars. The application of presented here analytic and numerical results to analysis of specific currently monitoring sources will be presented in forthcoming papers.

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